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Advances in Proof Theory

 Birkhäuser

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Preface

Advances in proof theory was the title of a symposium organized on the occasion of the 60th birthday of Gerhard Jäger. The meeting took place on December 13 and 14, 2013, at the University of Bern, Switzerland.

The aim of this symposium was to bring together some of the best specialists from the area of proof theory, constructivity, and computation and discuss recent trends and results in these areas. Some emphasis was put on ordinal analysis, reductive proof theory, explicit mathematics and type-theoretic formalisms, as well as abstract computations.

Gerhard Jäger has devoted his research to these topics and has substantially advanced and shaped our knowledge in these fields.

The program of the symposium was as follows:

Friday, December 13

Wolfram Pohlers: *From Subsystems of Classical Analysis to Subsystems of Set Theory: A personal account*

Wilfried Buchholz: *On the Ordnungszahlen in Gentzen's First Consistency Proof*

Andrea Cantini: *About Truth, Explicit Mathematics and Sets*

Peter Schroeder-Heister: *Proofs That, Proofs Why, and the Analysis of Paradoxes*

Roy Dyckhoff: *Intuitionistic Decision Procedures since Gentzen*

Grigori Mints: *Two Examples of Cut Elimination for Non-Classical Logics*

Rajeev Goré: *From Display Calculi to Decision Procedures via Deep Inference for Full Intuitionistic Linear Logic*

Pierluigi Minari: *Transitivity Elimination: Where and Why*

Saturday, December 14

Per Martin-Löf: *Sample Space-Event Time*

Anton Setzer: *Pattern and Copattern Matching*

Helmut Schwichtenberg: *Computational Content of Proofs Involving Coinduction*

Michael Rathjen: *When Kripke-Platek Set Theory Meets Powerset*

Stan Wainer: *A Miniaturized Predicativity*

Peter Schuster: *Folding Up*

Solomon Feferman: *The Operational Perspective*

This volume comprises contributions of most of the speakers and represents the wide spectrum of Gerhard Jäger's interests. We deeply miss Grisha Mints who planned to contribute to this Festschrift.

We acknowledge gratefully the financial support of Altonaer Stiftung für philosophische Grundlagenforschung, Bürgergemeinde Bern, Swiss Academy of Sciences, Swiss National Science Foundation, and Swiss Society for Logic and Philosophy of Science. We further thank the other members of the program committee, namely Roman Kuznets, George Metcalfe, and Giovanni Sommaruga.

For the production of this volume, we thank the editors of the *Progress in Computer Science and Applied Logic (PCS)* Series, the staff members of Birkhäuser/Springer Basel, and the reviewers of the papers of this volume.

We dedicate this Festschrift to Gerhard Jäger and thank him for his great intellectual inspiration and friendship.

Lisbon
Bern
Bern
December 2015

Reinhard Kahle
Thomas Strahm
Thomas Studer

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