

# DEDUCTION CHAINS FOR LOGIC OF COMMON KNOWLEDGE

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Deduction chains represent a syntactic and in a certain sense constructive method for proving completeness of a formal system  $S$  with respect to the class of  $S$ -structures. Given a formula  $A$ , the deduction chains of  $A$  are built up by systematically decomposing  $A$  into its subformulae. In the case where  $A$  is a valid formula, the decomposition yields a (usually cut-free) proof of  $A$  in the system  $S$ . If  $A$  is not valid, the decomposition produces a countermodel for  $A$ . The method of deduction chains was originally devised by Schütte and is mainly used in the context of first order logic and semiformal systems for various forms of arithmetic. Schütte [2] has also adapted the method to modal logic. In the current study, we extend this approach to a semiformal system for the Logic of Common Knowledge, as studied by Alberucci and Jäger [1]. The presence of fixed point constructs in this logic leads to potentially infinite-length deduction chains of a non-valid formula, in which case fairness of decomposition requires special attention. An adequate order of decomposition also plays an important role in the reconstruction of the proof of a valid formula from the set of its deduction chains.

## REFERENCES

- [1] L. ALBERUCCI and G. JÄGER, *About cut elimination for logics of common knowledge*, to appear in *Annals of Pure and Applied Logic*.
- [2] K. SCHÜTTE, *Vollständige Systeme modaler und intuitionistischer Logik*, *Ergebnisse der Mathematik und ihrer Grenzgebiete*, vol. 42, Springer-Verlag, 1968.

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