Infinitary Systems for the Modal mu-Calculus

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Our work is concerned with the proof theoretic relationship between two infinitary deductive systems for the propositional modal μ-calculus. The μ-calculus is defined by the addition of least and greatest fixed point operators to (multi-)modal logic. This results in a great increase in the expressive power: the modal μ-calculus includes most of the languages used for program verification. However, it is also much more difficult to present complete deductive systems for the modal μ-calculus since its language allows for arbitrary nestings of (possibly interleaved) fixed points.

There are two approaches to define infinitary axiomatizations for the μ-calculus. The first approach is to make use of so-called ω rules that have infinitely many premises to ensure that a fixed point is a least (or greatest) one. $\mathsf{T}^{\omega}_{\mu+}$ is such a system studied in [2]. There, completeness of $\mathsf{T}^{\omega}_{\mu+}$ is established by generalizing standard techniques for modal logics.

A second approach is to define a deductive system $\mathsf{T}^{\mathsf{pre}}_{\mu}$ such that in a proof search procedure fixed points are simply unfolded (which corresponds to closure of fixed points). This results in a so-called preproof which may have infinitely long branches. A global condition is then added which (roughly) says that in every such an infinite branch, there must be an outermost greatest fixed point unfolded infinitely many often. Such a system is proposed for example in [1].

We show that given a $\mathsf{T}^{\omega}_{\mu+}$ proof of a formula A of the μ-calculus, one can explicitly construct a $\mathsf{T}^{\mathsf{pre}}_{\mu}$ proof of A. This provides:

1. a completeness proof of $\mathsf{T}^{\mathsf{pre}}_{\mu}$ since $\mathsf{T}^{\omega}_{\mu+}$ is complete,
2. a soundness proof of $\mathsf{T}^{\omega}_{\mu+}$ since $\mathsf{T}^{\mathsf{pre}}_{\mu}$ is sound,
3. a proof-theoretic proof of the finite model property of the μ-calculus since the canonical counter model construction for $\mathsf{T}^{\omega}_{\mu+}$ can now be finitized.

References

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