COMPUTING WITH COMMON KNOWLEDGE

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Abstract

There are several situations in which *common knowledge* becomes important. Among them we have games, social behaviour, and natural language. In this paper we present a multi-agent framework in which the agent are able to use not only their own private knowledge bases bu also the common knowledge they share with other agents. This approach is based on *Aumann structures* and the language is a simplified version of full common knowledge. The restricted language retains enough expressive power as to be able to deal with the classical "muddy children" example while being relatively simple [1].

1 Introduction

When we say that a group of agents share *common knowledge* [2, 5, 6], we mean a situation in which everyone in the group knows something, everyone knows that everyone knows, and so on. In multi-agent systems that occurs in situations where *epistemic knowledge* (i.e., knowledge about the knowledge of other agents) is relevant. Such situations include, for instance, natural language (there must be a common understanding of the meaning of words and expressions), some games like Bridge, or in simulations of social behaviour, in which the opinions of others may influence our acts [4]. In the case of one single agent, we may consider this as a instance of *introspective knowledge* (the agent ist conscious of its own knowledge.)

Our goal is to explore how an agent could complement its own knowledge with such epistemic knowledge to draw conclusions and take decisions. We will deal with a restricted form of common knowledge. We will thus have a relatively simple system with an acceptable expressive power. The agents have some private knowledge of the actual state of the world and they exchange communications with other agents, thus creating common knowledge between them. Besides, *public announcements* creates common knowledge among all agents. We show how the agents may draw conclusions not only from their own knowledge base, but also from the information they get about the knowledge other agents have. Public announcements introduce a dynamical aspect in our language, similar to the analised for instance in [3, 7].

Since we will be concerned with *knowledge* and not with *beliefs*, the different representations must be consistent with each other. Tipically, the common knowledge will be a subset of the private knowledge.

The paper is organised as follows. In the section 2 we begin with a simple motivating example to provide a flavour of what is going on. Most examples are taken from [2], and are already classical in the literature on epistemic systems. In section 3 we give the syntax and the semantics of the language we work with. Section 4 contains some examples of the use of the system, and section 5 contains the conclusions and some suggestions to extend this approach.

2 A First Example

In this section we will consider the most simple case, in which we have a set of agents with some epistemic knowledge and there are public announcements, that are of the form ! φ, where φ is some formula. An announcement is public not only in the sense that everyone receives the information, but also in the sense that everyone knows that everyone receives the information, everyone knows that everyone knows that everyone receives the information, and so on. It would be therefore tempting to consider that as a result of the announcement φ becomes common knowledge for everyone. This is not necessarily the case, as the example below shows. The example we introduce is the classical "muddy children" puzzle, which is a classical example in the literature about common knowledge [2, 5].

Example (Minimalist Muddy Children.) Assume there are two agents (the children) 1 and 2 and that the predicates $d1$ and $d2$ specify whether they are clean or dirty. Assume that they both can see whether the other one is dirty, but no one can see herself, and both children are aware of this.

Now assume that somebody tells them that at least one of them is dirty. At first sight, this introduces no modification, since both children already knew that. But now *they know that the other one knows*.

If they now exclaim at the same time that they do not kow whether they are dirty or not, this couldbe considered a kind of public announcement. Nevertheless, this is not common knowledge since now both children know that they are dirty (otherwise the other one would know that she is the one) and thus the public announcement has become

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false because of the very fact that it was made.

Notice that if the first announcement would not have occurred, the second one would not be enough for the children to know their state. The reason why the announcement in the example has not become common knowledge is that it consists of non-monotonic information (i.e., information about ignorance), what introduces a kind of "instability." In the next sections we will try to formalise the syntax and semantics of a language that will enable us to deal with similar problems. announcements.

3 The Language

3.1 Syntax of the Language

Our language will describe the epistemic state of the agents, i.e., we want to express what an agent knows and what a group of agents has common knowledge about. We will thus distinguish between *objective formulæ* (those not involving common knowledge), *epistemic formulæ* (those describing common knowledge), and *communications* (either public announcements or private communications.) We will assume the existence of a set of propositional symbols Π, whose elements will be represented by $p, q, \ldots$, and a set of agents A whose elements will be represented by $i, j, \ldots$. The syntax of objective formulæ, which we will denote by OBJ, is given by the following grammar:

$$\mathsf{OBJ} ::= p \mid \neg p \mid \mathsf{OBJ} \wedge \mathsf{OBJ} \mid \mathsf{OBJ} \vee \mathsf{OBJ}$$

The syntax of epistemic formulæ, which we will denote by EPI, is given by the following grammar, where $G \subseteq A$ is a group of agents:

$$\begin{array}{lcl} \mathsf{EPI} & ::= & \mathsf{C}_G\mathsf{OBJ} \mid \mathsf{KW}_G\mathsf{OBJ} \mid \mathsf{NKW}_G\mathsf{OBJ} \mid \\ & & \mathsf{EPI} \wedge \mathsf{EPI} \mid \neg\mathsf{EPI} \end{array}$$

Note that we do not have formulæ of the form $\mathsf{C}_{G_1}\mathsf{C}_{G_2}\varphi$. This is because such formulæ could appear only as a result of private communications. Due to the restriction we impose on the communications, such formulæ cannot be constructed. We will point out in the last section how to treat such cases. In the case of common knowledge among all agents ($G = A$), we will omit the subindex ($\mathsf{C}_A\varphi$ will be simply written as $\mathsf{C}\varphi$.) Finally, we have *communications*, which may be either *public announcement* or *private communications*. A communication has the form $!^i_G\varphi$, where $i \in A$ is the *emitter* of the language, $G \subseteq A$ is the *receiver*, and φ is a restricted epistemic formula. The meaning of $!^i_G\varphi$ is that agent i communicates φ to all agents in group G. We only allow formulæ occurring within communications to be of the following form:

$$\begin{array}{lcl} \mathsf{COM} & ::= & !^i_G\mathsf{OBJ} \mid !^i_G\mathsf{KW}_{G'}\mathsf{OBJ} \mid !^i_G\mathsf{NKW}_i\mathsf{OBJ} \mid \\ & & !\ \mathsf{NKW}_i\mathsf{OBJ} \end{array}$$

Observe that the ignorance (NKW) of an agent i may only be communicated by a public announcement or by the agent; no agent is allowed to communicate the ignorance of other agents. The reason for this is not that we want our agents to behave politely to each other, but to avoid inconsistency. The reason will be clear later on. We will also see that this restriction limits the complexity of the epistemic formulæ that may occur. Public announcements have the form $!\ \varphi$ because there is no emitter and the receiver is A, the set of all agents.

The semantics of the language will be formalised in the next section. Informally, a formula $\mathsf{C}_G\varphi$ means that every agent in group G knows φ, and everyone knows that everyone know φ and so on. The formula $\mathsf{KW}_G\varphi$ means that every agent in group G knows whether φ holds or not. Note that in this case, it is the same to write $\mathsf{KW}_G\varphi$ or $\mathsf{KW}_G\neg\varphi$. The formula $\mathsf{NKW}_G\varphi$ is simply the nefation of the previous one: no agent in group G knows whether φ holds or not. we do not have a special operator for the knowledge of one single agent, since we have only common knowledge; individual common knowledge is simply introspective knowledge.

3.2 Semantics of the Language

The semantics of the language will be given in terms of *epistemic structures*, which are very similar to Aumann structures [1].

As before, we assume the existence of a set A of agents, a set Π of propositional symbols, and a set S of states. We assume further that there is a mapping $V : \Pi \mapsto 2^S$, called a *valuation* for Π, whose intuitive meaning is that $V(p)$ gives the set of states where p holds for each proposition $p \in \Pi$.

Definition (Partitions, Finer Partitions, Coarser Partitions.) Let S be a set. Then a *partition* of S is a set $\mathcal{P}(S) = \{\mathcal{P}_1, \ldots, \mathcal{P}_m\}$ of subsets of S such that $\bigcup_{i=1}^m \mathcal{P}i = S$ and $i \neq j$ implies that $\mathcal{P}_i \cap \mathcal{P}_j = \emptyset$. Given an element $s \in S$, we denote by $\mathcal{P}(s)$ the partition $\mathcal{P}_i$ such that $s \in \mathcal{P}_i$. A partition $\mathcal{P}_A(S)$ is *finer* than a partition $\mathcal{P}_B(S)$ (correspondingly $\mathcal{P}_B(S)$ is *coarser* than $\mathcal{P}_A(S)$) if for all $s \in S$ it is the case that $\mathcal{P}_A(s) \subseteq \mathcal{P}_B(s)$. Given two sets S and $S' \subseteq$, we may also say that a partition $\mathcal{P}_A(S')$ is *finer* than a partition $\mathcal{P}_B(S)$ if for all $s \in S'$ it is the case that $\mathcal{P}_A(s) \subseteq \mathcal{P}_B(s)$.

If S is a set of states, we will use partitions of $S' \subseteq S$ to group the states that are indistinguishable for an agent.

Example. Assume that a state is fully described by two propositions p and q. There are thus four possible states, namely $s_0 = (\neg p, \neg q)$, $s_1 = (\neg p, q)$, $s_2 = (p, \neg q)$ and $s_3 = (p, q)$. If an agent knows p and $\neg q$, its corresponding partition is $\{\{s_3\}\}$. If it knows nothing, its partition is $\{\{s_0, s_1, s_2, s_3\}\}$. If it just knows $\neg q$ its partition is $\{\{s_0, s_2\}\}$. Finally, if we know that the agent knows p or knows $\neg p$, its partition is $\{\{s_0, s_1\}, \{s_2, s_3\}\}$.

If we see the example above, we can give the intuitive interpretation that a finer partition has at least as much information as a coarser one: the more knowledge an agent has, the less states it considers possible. When it has absolute certainty, only one state (the "right" one) is taken into account. When it has absolute ignorance, all states are considered equally possible. Observe also that, unlike Aumann structures, our partitions range over sets that are not necessarily the set of all states. This is because we eliminate states that are no longer considered possible.

Definition. Let A be a set of agents, $G \subseteq A$, S a set of states, and let $S_I \subseteq S_O \subseteq S$[2]. An *epistemic state* for A, G, and S is a pair $\mathbb{E}_G = \{E_I, E_O\}$ such that

- E_I is a set $E_{I,j}$ of partitions of S_I such that for all $j \in G$ there is a partition $E_{I,j} \in E_I$.
- E_O is a set $E_{O,i}$ of partitions of S_O such that for all $i \in A \backslash G$ there is a partition $E_{O,i} \in E_O$.

Given a partition $\mathbb{E}_G$ we use the notation $I(\mathbb{E}_G)$ for its corresponding set S_I and $O(\mathbb{E}_G)$ for its corresponding set S_O.

An epistemic state $\mathbb{E}_G$ will represent the common knowledge a group G of agents (the "insiders") shares about the knowledge of all others. The agents of group $A \backslash G$, that have not access to $\mathbb{E}_G$, are the "outsiders." In the case of common knowledge among all agents in A, all agents are insiders. All insiders having access to an epistemic state have of course access to all partitions, either of other insiders or of outsiders.

Example. Let $A = \{1, 2, 3\}$, $S = \{s_0, s_1, s_2, s_3\}$ and let the states be defined by $s_0 = \{\neg p, \neg q\}$, $s_1 = \{\neg p, q\}$, $s_2 = \{p, \neg q\}$, and $s_3 = \{p, q\}$. Assume further that it is common knowledge among all agents that $\mathsf{KW}_1 p$, $\mathsf{NKW}_1 q$, $\mathsf{KW}_2 q$, $\mathsf{NKW}_2 p$, $\mathsf{KW}_3 q$, and $\mathsf{NKW}_3 p$. Thus we have that $\mathbb{E}$ is the following epistemic state:

agent	partition
1	$\{\{s_0, s_1\}, \{s_2, s_3\}\}$
2	$\{\{s_0, s_2\}, \{s_1, s_3\}\}$
3	$\{\{s_0, s_2\}, \{s_1, s_3\}\}$

What would happen if 1 and 2 exchange their private knowledge? Then both would know that state s_3 is the actual one, and for 3 nothing would have changed. Since there was no public announcement, the common knowledge of all agents remains unchanged. But there is a new epistemic state $\mathbb{E}_{\{1,2\}}$ which is the following one:

	agent	partition
E_I	1	$\{\{s_3\}\}$
	2	$\{\{s_3\}\}$
E_O	3	$\{\{s_0, s_2\}, \{s_1, s_3\}\}$

Observe that agents 1 and 2 (the insiders) have a smaller subset of states than agent 3 (the outsider), because they are the only ones in $\mathbb{E}_{\{1,2\}}$ that may take advantage of new information.

Definition (Epistemic Structures.) Let us assume that we have a set of agents $A = \{1, \ldots, n\}$, a set of atomic propositions Π, and a set of states S. An *epistemic structure* on A, S, and Π is a triple $\mathfrak{E} = \langle \mathcal{K}, \mathcal{E}, V \rangle$ where $\mathcal{K} = \{K_1, \ldots, K_n\}$ is a set of subsets of S such that for each agent i there is a set K_i, $\mathcal{E} = \{\mathbb{E}_G \mid G \in 2^S\}$ is a set of epistemic states such that for each non-empty subset G of A there is an epistemic state $\mathbb{E}_G \in \mathcal{E}$, and $V : \Pi \mapsto 2^S$ is a valuation.

Epistemic structures will suffice to express the general state of the system with the restrictions already stated. The set $\mathcal{K}$ will contain the private knowledge of each agent, expressed as a set of states which are undistinguishable for it. Before establishing the semantics of our system we need some further definitions.

Definition (Satisfiability Relation.) Let us assume that we have a set of states $S = \{s_1, \ldots, s_m\}$, a set of agents $A = \{1, \ldots, n\}$, and a set of atomic propositions Π. Let us further assume we have a valuation $V : \Pi \mapsto 2^S$. Then we define the *satisfiability relation* as follows:

- For a state $s_i \in S$ and an objective formula φ the satisfiability relation is defined inductively:
  - If $p \in \Pi$, $s \models p$ iff $s \in V(p)$ and $s \models \neg p$ iff $s \notin V(p)$.
  - $s \models \psi \wedge \zeta$ iff $s \models \psi$ and $s \models \zeta$
  - $s \models \psi \vee \zeta$ iff $s \models \psi$ or $s \models \zeta$
- For a set of states $E \subseteq S$ and an objective formula φ, we have:
  - If $p \in \Pi$, $E \models p$ iff $s \in V(p)$ for all $s \in E$ and $E \models \neg p$ iff $s \notin V(p)$ for all $s \in E$.
  - $E \models \psi \wedge \zeta$ iff $E \models \psi$ and $E \models \zeta$
  - $E \models \psi \vee \zeta$ iff $E \models \psi$ or $E \models \zeta$
- For two sets of agents G and G' such that $G' \subseteq G \subseteq A$, an epistemic state $\mathbb{E}_G = \{E_1, \ldots, E_n\}$ and an objective formula φ we have:
  - $\mathbb{E}_G \models \mathsf{C}_G \varphi$ iff for all $s \in \bigcup \mathbb{E}_G$ it is the case that $s \models \varphi$.
  - $\mathbb{E}_G \models \mathsf{KW}_{G'} \varphi$ iff for all $E_i \in \mathbb{E}_G$ such that $i \in G'$, we have that for all partitions $\mathcal{P}_j \in E_i$ either $\mathcal{P}_j \models \varphi$ or $\mathcal{P}_j \models \neg\varphi$.
  - $\mathbb{E}_G \models \mathsf{NKW}_{G'} \varphi$ iff for all $E_i \in \mathbb{E}_G$ such that $i \in G'$, we have that for all partitions $\mathcal{P}_j \in E_i$ neither $\mathcal{P}_j \models \varphi$ nor $\mathcal{P}_j \models \neg\varphi$.

Communications will introduce a dynamical element, since the epistemic structures will change whenever an announcement is done. We assume that communications are *admissible*, in the sense that any agent can only communicate information that it knows (in other words, agents have no immagination; they cannot "invent" anything.) Definition (Transformation Function.) Let A, S, and Π as above. The operator τ transforms an epistemic structure $\mathfrak{E} = \langle \mathcal{K}, \mathcal{E}, V \rangle$ on A and S into another one when a communication occurs. Assuming a an epistemic state $\mathbb{E}_G \in \mathcal{E}$ and a communication $!^i_G \varphi$ for some $i \in G$, we have:

[2] S_I stays for "insiders' states" and S_O for "outsiders' states."

1. If $\varphi = p$, then all states $s \in I(\mathbb{E}_G)$ such that $s \notin V(p)$ are eliminated from from $I(\mathbb{E}_G)$. If $\varphi = \neg p$, then all states $s \in I(\mathbb{E}_G)$ such that $s \in V(p)$ are eliminated from $I(\mathbb{E}_G)$.

2. If $\varphi = \psi \wedge \zeta$, all states $s \in I(\mathbb{E}_G)$ such that either $s \models \neg\psi$ or $s \models \neg\zeta$ are eliminated from $I(\mathbb{E}_G)$.

3. If $\varphi = \psi \vee \zeta$, all states $s \in I(\mathbb{E}_G)$ such that either $s \models \neg\psi$ and $s \models \neg\zeta$ are eliminated from $I(\mathbb{E}_G)$.

4. If $\varphi = \mathsf{KW}_{G'}\psi$, with $G' \subseteq G$, then for all $i \in G'$ we refine the partition $E_i \in \mathbb{E}_G$ by replacing each $\mathcal{P}_j \in E_i$ by $\mathcal{P}_{j,1}$ and $\mathcal{P}_{j,2}$, two sets of states satisfying the following conditions:

    - $\mathcal{P}_{j,1} \cup \mathcal{P}_{j,2} = \mathcal{P}_j$.
    - $\mathcal{P}_{j,1} \cap \mathcal{P}_{j,2} = \emptyset$
    - $\mathcal{P}_{j,1} \models \psi$.
    - $\mathcal{P}_{j,2} \models \neg\psi$.

5. If $\varphi = \mathsf{NKW}_i\psi$, then all states belonging to partitions $\mathcal{P}_j \in E_i$ such that either $\mathcal{P}_j \models \psi$ or $\mathcal{P}_j \models \neg\psi$ are eliminated from $\mathbb{E}_G$.

6. (Inheritance Condition): the states that have been eliminated from an epistemic state $\mathbb{E}_G$ must also be eliminated from all epistemic states $\mathbb{E}_{G'}$ with $G' \subset G$ and from all sets $K_i \in \mathcal{K}$ with $i \in G$.

In the simplified case in which only public announcements are allowed, we have that rule 4 may be replaced by the following: If $\varphi = \mathsf{KW}_i\psi$, then all states belonging to partitions $\mathcal{P}_j \in E_i$ such that neither $\mathcal{P}_j \models \psi$ nor $\mathcal{P}_j \models \neg\psi$ are eliminated from $\mathbb{E}_G$.

Now we are ready to define the semantics of our language.

Definition. (Semantics of the Language.) Let $A = \{1, \ldots, n\}$ be a set of agents, S, a set of states, Π a set of atomic propositions. Assume that each agent i owns some formulæ that restrict the set of states it considers possible to $S_i \subseteq S$. Then a *model* for this system is an infinite tree of epistemic structures $\mathfrak{E}_j$ on A, S, and Π such that:

- The root of the tree is the epistemic structure $\mathfrak{E}_0 = \langle \mathcal{K}_0, \mathcal{E}_0, V\rangle$ such that for all $K_i \in \mathcal{K}_0$ we have that $K_i = S_i$ and all the epistemic structures in $\mathcal{E}_0$ represent absolute ignorance (i.e., their partitions are singletons with all the states.)

- For all epistemic structures $\mathfrak{E}$ and all admissible communications $c = !^i_G\varphi$ (or $c =! \ \varphi$) in $\mathfrak{E}$ there is a successor node $\mathfrak{E}'$ with $\mathfrak{E}' = \tau(\mathfrak{E}, c)$.

We will see in the next section that all this is simpler than it looks.

4 Examples

Example (Muddy Children Revisited.) Let us assume now the existence of three agents (the children), $A = \{1, 2, 3\}$ and three atomic propositions $\Pi = \{d1, d2, d3\}$ respectively stating that "child i is dirty" for each child. Assume further that each child can see whether the others are dirty or not, but does not know her own situation. We have the following state table, where the actual state is denoted with a bullet:

	st	$d1$	$d2$	$d3$
	0	0	0	0
	1	0	0	1
	2	0	1	0
	3	0	1	1
	4	1	0	0
	5	1	0	1
•	**6**	1	1	0
	7	1	1	1

The situation at the beginning is that everyone sees whether the others are dirty or not, but no one is aware of her own situation. We have thus

$$K_1 = \{3, 6\}, K_2 = \{4, 6\}, K_3 = \{6, 7\}$$

Since, for instance, 1 cannot distinguish between states **3** and **6**. So far we have no common knowledge, since no public announcements have been made. Assume now that the following public announcement is broadcast:

$$! \ \mathsf{KW}_1 d2 \wedge \mathsf{KW}_1 d3 \wedge \mathsf{KW}_2 d1 \wedge \mathsf{KW}_2 d3 \wedge \mathsf{KW}_3 d1 \wedge \mathsf{KW}_3 d2$$

Now we do have common knowledge, which is represented by the following epistemic structure $\mathbb{E}$:

agent	partition
1	$\{\{0,4\},\{1,5\},\{2,6\},\{3,7\}\}$
2	$\{\{0,2\},\{1,3\},\{4,6\},\{5,7\}\}$
3	$\{\{0,1\},\{2,3\},\{4,5\},\{6,7\}\}$

The private knowledge of the agents does not change, since $\mathbb{E}$ neither eliminates states nor refines on individual information. Now we get the new public announcement

$$! \ d1 \vee d2 \vee d3$$

("At least one of you is dirty!") This amounts to eliminating state **0**. We get thus the following transformed epistemic state:

agent	partition
1	$\{\{4\},\{1,5\},\{2,6\},\{3,7\}\}$
2	$\{\{2\},\{1,3\},\{4,6\},\{5,7\}\}$
3	$\{\{1\},\{2,3\},\{4,5\},\{6,7\}\}$

Again, the private knowledge remains unaltered, since state **0** was not considered for any individual agent. Then, what has changed? Let us consider the private knowledge of 2: there are two states, **4** and **6** which are equally possible and undistinguishable. But if state **4** were the right one, child 2 knows that child 1 would have certainty. Now we get the following public annoucement:

$$!\mathsf{NKW}_1 d1 \wedge \mathsf{NKW}_2 d2 \wedge \mathsf{NKW}_3 d3$$

Now all "certainty" states **1**, **2**, and **4** are eliminated from $\mathbb{E}$, yielding:

agent	partition
1	$\{\{5\}, \{6\}, \{3, 7\}\}$
2	$\{\{3\}, \{6\}, \{5, 7\}\}$
3	$\{\{3\}, \{5\}, \{6, 7\}\}$

Now the private knowledge of the children is changed according to the inheritance rule:

$$K_1 = \{6\}, K_2 = \{6\}, K_3 = \{6, 7\}$$

This is not surprising, because of what we argued before. Assume now that 1 and 2 issue the communications

$$!^1_{\{1,2,3\}}\mathsf{KW}_1 d1$$

$$!^2_{\{1,2,3\}}\mathsf{KW}_2 d2$$

Here we have two possibilities. Assume that no private communications are allowed. Then we have to eliminate states belonging to subsets $\mathcal{P}_j \in E_1$ such that neither $\mathcal{P}_j \models d1$ nor $\mathcal{P}_j \models \neg d1$ and all states belonging to subsets $\mathcal{P}_k \in E_2$ such that neither $\mathcal{P}_k \models d2$ nor $\mathcal{P}_k \models \neg d2$. The only subsets in this situation are $\{3, 7\} \in E_1$ and $\{5, 7\} \in E_2$, and after elimination of states **3**, **5**, and **7** all agents converge to the actual state **6**. If we are in the more general setting in which we allow private communications, we have to partition all subsets $\mathcal{P}_j$ of E_1 and E_2 are two subsets according to the rules we saw before. This operation yields:

agent	partition
1	$\{\{3\}, \{5\}, \{6\}, \{7\}\}$
2	$\{\{3\}, \{5\}, \{6\}, \{7\}\}$
3	$\{\{3\}, \{5\}, \{6, 7\}\}$

This is the finest epistemic state we may have without 3 being aware of the real state. Note that, in contrast to the classical setting of muddy children, where only public announcements are allowed, here 3 cannot infer her state. This is because the knowledge of 1 and 2 could have been the result of private communications of which 3 is unaware.

We have seen in the last example how public announcement works. Now we will consider the case in which private communications occur.

Example (Cheating Muddy Children.) Assume the same scenario as in the last example until just after the communication that at least one of them is dirty. Recall we have the following common knowledge epistemic state:

agent	partition
1	$\{\{4\}, \{1, 5\}, \{2, 6\}, \{3, 7\}\}$
2	$\{\{2\}, \{1, 3\}, \{4, 6\}, \{5, 7\}\}$
3	$\{\{1\}, \{2, 3\}, \{4, 5\}, \{6, 7\}\}$

Assume further that 1 whispers to 2 "You are the dirty!" We would have:

$$!^1_2 d2$$

Thus, a new epistemic state $\mathbb{E}_{\{1,2\}}$ where 1 and 2 are the insiders is generated, namely:

agent	partition
1	$\{\{2, 6\}, \{3, 7\}\}$
2	$\{\{2\}, \{3\}, \{6\}, \{7\}\}$
3	$\{\{1\}, \{2, 3\}, \{4, 5\}, \{6, 7\}\}$

Observe that here 2 has already because of the inheritance condition enough elements to know that the right state is **6**. For 3 nothing has changed, since she did not overhear the communication, and for 1 nothing has changed either, except that she knows that 2 knows, since all subsets in the partition of 2 in $\mathbb{E}_{\{1,2\}}$ are singletons.

5 Conclusions and Future Work

We have presented a multi-agent system in which the agents may extract conclusions based not only in their own databases but also in epistemic knowledge. The framework is loosely based on Aumann structures [1] and some earlier work by Baltag et alii [2, 3]. Some restrictions have been made to keep the problem manageable and the semantics relatively simple.

In order to accept arbitrary formulæ in communications, the semantics must be more complex. In a way, we cannot avoid using the traditional Kripke structures as in [5] or something that is equivalent. This is so because a communication like $!^i_{G1}\mathsf{C}_{G2}\varphi$ could be interpreted as giving the group $G1$ access to *a copy* of $\mathbb{E}_{G2}$; we could not give access to $\mathbb{E}_{G2}$ because the message was sent at a given moment in time; $G2$ should not know whether modifications to $\mathbb{E}_{G2}$ occur *after* the message.

The complexity of the system thus presented may be in the worst case exponential. The worst case occurs when many private communications with different emitters and receivers occur. For each communication $!^i_G\varphi$ a new epistemic state $\mathbb{E}_{G\cup\{i\}}$ is generated (if the epistemic state exists already, it is just refined.) Thus, the generalisation to communications with arbitrary formu;æ might render the problem unmanageable. On the other hand, restriction of communications to public announcements considerably simplify the problem, since only one epistemic state representing the common knowledge of all agents is needed.

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